Year 6 guidance

Ready-to-progress criteria

Year 5 conceptual prerequesite	Year 6 ready-to-progress criteria	Key stage 3 applications
Understand the relationship between powers of 10 from 1 hundredth to 1,000 in terms of grouping and exchange (for example, 1 is equal to 10 tenths) and in terms of scaling (for example, 1 is ten times the size of 1 tenth).	6NPV–1 Understand the relationship between powers of 10 from 1 hundredth to 10 million, and use this to make a given number 10, 100, 1,000, 1 tenth, 1 hundredth or 1 thousandth times the size (multiply and divide by 10, 100 and 1,000).	Understand and use place value for decimals, measures, and integers of any size. Interpret and compare numbers in standard form $A \times 10^n 1 \leq A < 10$, where n is a positive or negative integer or zero.
Recognise the place value of each digit in numbers with units from thousands to hundredths and compose and decompose these numbers using standard and non-standard partitioning.	6NPV–2 Recognise the place value of each digit in numbers up to 10 million, including decimal fractions, and compose and decompose numbers up to 10 million using standard and non-standard partitioning.	Understand and use place value for decimals, measures, and integers of any size. Order positive and negative integers, decimals, and fractions. Use a calculator and other technologies to calculate results accurately and then interpret them appropriately.
Reason about the location of numbers between 0.01 and 9,999 in the linear number system. Round whole numbers to the nearest multiple of 1,000, 100 or 10, as appropriate. Round decimal fractions to the nearest whole number or nearest multiple of 0.01	6NPV–3 Reason about the location of any number up to 10 million, including decimal fractions, in the linear number system, and round numbers, as appropriate, including in contexts.	Order positive and negative integers, decimals, and fractions; use the number line as a model for ordering of the real numbers; use the symbols =, ≠, <, >, ≤, ≥ Round numbers and measures to an appropriate degree of accuracy (for example, to a number of decimal places or significant figures). Use approximation through rounding to estimate answers and calculate possible resulting errors expressed using inequality notation $a < x \leq b$

Year 5 conceptual prerequesite	Year 6 ready-to-progress criteria	Key stage 3 applications
Divide 1000, 100 and 1 into 2, 4, 5 and 10 equal parts, and read scales/number lines with 2, 4, 5 and 10 equal parts.	6NPV–4 Divide powers of 10, from 1 hundredth to 10 million, into 2, 4, 5 and 10 equal parts, and read scales/number lines with labelled intervals divided into 2, 4, 5 and 10 equal parts.	Use standard units of mass, length, time, money, and other measures, including with decimal quantities. Construct and interpret appropriate tables, charts, and diagrams.
Be fluent in all key stage 2 additive and multiplicative number facts (see Appendix: number facts fluency overview) and calculation. Manipulate additive equations, including applying understanding of the inverse relationship between addition and subtraction, and the commutative property of addition. Manipulate multiplicative equations, including applying understanding of the inverse relationship between multiplication and division, and the commutative property of multiplication.	6AS/MD–1 Understand that 2 numbers can be related additively or multiplicatively, and quantify additive and multiplicative relationships (multiplicative relationships restricted to multiplication by a whole number).	Understand that a multiplicative relationship between 2 quantities can be expressed as a ratio or a fraction. Express 1 quantity as a fraction of another, where the fraction is less than 1 and greater than 1. Interpret mathematical relationships both algebraically and geometrically. Interpret when the structure of a numerical problem requires additive, multiplicative or proportional reasoning.
Make a given number (up to 9,999, including decimal fractions) 10, 100, 1 tenth or 1 hundredth times the size (multiply and divide by 10 and 100). Apply place-value knowledge to known additive and multiplicative number facts (scaling facts by 10, 100, 1 tenth or 1 hundredth). Manipulate additive equations. Manipulate multiplicative equations.	6AS/MD–1 Use a given additive or multiplicative calculation to derive or complete a related calculation, using arithmetic properties, inverse relationships, and place-value understanding.	Recognise and use relationships between operations including inverse operations. Use algebra to generalise the structure of arithmetic, including to formulate mathematical relationships. Understand and use standard mathematical formulae; rearrange formulae to change the subject.

Year 5 conceptual prerequesite	Year 6 ready-to-progress criteria	Key stage 3 applications
Recall multiplication and division facts up to 12 × 12. Apply place-value knowledge to known additive and multiplicative number facts.	6AS/MD–3 Solve problems involving ratio relationships.	Use ratio notation, including reduction to simplest form. Divide a given quantity into 2 parts in a given part:part or part:whole ratio; express the division of a quantity into 2 parts as a ratio.
Be fluent in all key stage 2 additive and multiplicative number facts and calculation. Manipulate additive equations. Manipulate multiplicative equations. Find a fraction of a quantity.	6AS/MD–4 Solve problems with 2 unknowns.	Reduce a given linear equation in two variables to the standard form $y = mx + c$; calculate and interpret gradients and intercepts of graphs of such linear equations numerically, graphically and algebraically. Use linear and quadratic graphs to estimate values of y for given values of x and vice versa and to find approximate solutions of simultaneous linear equations.
Recall multiplication and division facts up to 12 × 12 . Find factors and multiples of positive whole numbers, including common factors and common multiples. Find equivalent fractions and understand that they have the same value and the same position in the linear number system.	6F–1 Recognise when fractions can be simplified, and use common factors to simplify fractions.	Use the concepts and vocabulary of prime numbers, factors (or divisors), multiples, common factors, common multiples, highest common factor, lowest common multiple, prime factorisation, including using product notation and the unique factorisation property. Simplify and manipulate algebraic expressions by taking out common factors.

Year 5 conceptual prerequesite	Year 6 ready-to-progress criteria	Key stage 3 applications
Recall multiplication and division facts up to 12×12. Find factors and multiples of positive whole numbers. Find equivalent fractions. Reason about the location of fractions and mixed numbers in the linear number system.	6F–2 Express fractions in a common denomination and use this to compare fractions that are similar in value.	Order positive and negative integers, decimals and fractions. Use the 4 operations, including formal written methods, applied to integers, decimals, proper and improper fractions, and mixed numbers, all both positive and negative. Use and interpret algebraic notation, including: a/b in place of $a \div b$ coefficients written as fractions rather than as decimals.
Reason about the location of fractions and mixed numbers in the linear number system. Find equivalent fractions.	6F–3 Compare fractions with different denominators, including fractions greater than 1, using reasoning, and choose between reasoning and common denomination as a comparison strategy.	Order positive and negative integers, decimals, and fractions; use the number line as a model for ordering of the real numbers; use the symbols $=, \neq, <, >, \leq, \geq$
Find the perimeter of regular and irregular polygons. Compare angles, estimate and measure angles in degrees (°) and draw angles of a given size. Compare areas and calculate the area of rectangles (including squares) using standard units.	6G–1 Draw, compose, and decompose shapes according to given properties, including dimensions, angles and area, and solve related problems.	Draw shapes and solve more complex geometry problems (see Mathematics programmes of study: key stage 3 - Geometry and measures).

6NPV–1 Powers of 10

Understand the relationship between powers of 10 from 1 hundredth to 10 million, and use this to make a given number 10, 100, 1,000, 1 tenth, 1 hundredth or 1 thousandth times the size (multiply and divide by 10, 100 and 1,000).

6NPV–1 Teaching guidance

An understanding of the relationship between the powers of 10 prepares pupils for working with much larger or smaller numbers at key stage 3, when they will learn to read and write numbers in standard form (for example, $600{,}000{,}000 = 6 \times 10^8$).

Pupils need to know that what they learnt in year 3 and year 4 about the relationship between 10, 100 and 1,000 (see 3NPV–1 and 4NPV–1), and in year 5 about the relationship between 1, 0.1 and 0.01 (5NPV–1) extends through the number system. By the end of year 6, pupils should have a cohesive understanding of the whole place-value system, from decimal fractions through to 7-digit numbers.

Pupils need to be able to read and write numbers from 1 hundredth to 10 million, written in digits, beginning with the powers or 10, as shown below, and should understand the relationships between these powers of 10.

0 . 0 1	one hundredth
0 . 1	one tenth
1	one
1 0	ten
1 0 0	one hundred
1 , 0 0 0	one thousand
1 0 , 0 0 0	ten thousand
1 0 0 , 0 0 0	one hundred thousand
1 , 0 0 0 , 0 0 0	one million
1 0 , 0 0 0 , 0 0 0	ten million

Pupils should know that each power of 10 is equal to 1 group of 10 of the next smallest power of 10, for example 1 million is equal to 10 hundred thousands.

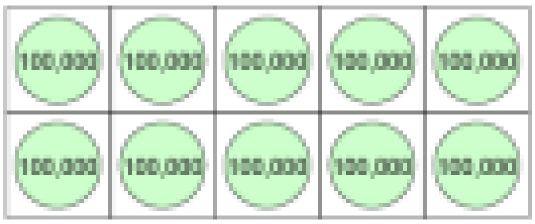


Figure 206: ten 100,000-value place-value counters in a tens frame

Language focus

"10 hundred-thousands is equal to 1 million."

Pupils should also understand this relationship in terms of scaling by 10 or one-tenth.

Language focus

"1,000,000 is 10 times the size of 100,000."

"100,000 is one-tenth times the size of 1,000,000."

Pupils must also understand the relationships between non-adjacent powers of 10 up to a scaling by 1,000 or 1 thousandth (or grouping of up to 1,000 of a given power).

Language focus

"10 thousands is equal to 10,000."

"10,000 is 10 times the size of 1,000."

"1,000 is one-tenth times the size of 10,000."

Pupils must also be able to write multiples of these powers of 10, including when there are more than 10 of given power of 10, for example, 18 hundred thousands is written as 1,800,000. Pupils should be able to restate the quantity in the appropriate power of 10, for example 18 hundred thousands is equal to 1 million 8 hundred thousand.

Once pupils understand the relationships between powers of ten, they should extend this to other numbers in the Gattegno chart. They must be able to identify the number that is 10, 100, 1,000, 1 tenth, 1 hundredth or 1 thousandth times the size of a given number, and associate this with multiplying or dividing by 10, 100 and 1,000. This will prepare pupils for multiplying by decimals in key stage 3, when they will learn, for example, that dividing by 100 is equivalent to multiplying by 0.01.

10,000,000	20,000,000	30,000,000	40,000,000	50,000,000	60,000,000	70,000,000	80,000,000	90,000,000
1,000,000	2,000,000	3,000,000	4,000,000	5,000,000	6,000,000	7,000,000	8,000,000	9,000,000
100,000	200,000	300,000	400,000	500,000	600,000	700,000	800,000	900,000
10,000	20,000	30,000	40,000	50,000	60,000	70,000	80,000	90,000
1,000	2,000	3,000	4,000	5,000	6,000	7,000	8,000	9,000
100	200	300	400	500	600	700	800	900
10	20	30	40	50	60	70	80	90
1	2	3	4	5	6	7	8	9
0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09

Figure 207: using the Gattegno chart to multiply and divide by 100

Language focus

"50,000 is 100 times the size of 500."
"500 multiplied by 100 is equal to 50,000."

"500 is one-hundredth times the size of 50,000."
"50,000 divided by 100 is equal to 500."

Pupils should recognise the inverse relationship between, for example making a number 100 times the size, and returning to the original number by making it one-hundredth times the size.

This understanding should then be extended to multiplicative calculations that involve numbers with *more* than one significant digit, extending what pupils learnt in 5MD–1 about multiplying and dividing by 10 and 100.

$1{,}659 \times 100 = 165{,}900$ $\quad$ $165{,}900 \div 100 = 1{,}659$

$21{,}156 \times 10 = 211{,}560$ $\quad$ $211{,}560 \div 10 = 21{,}156$

$47.1 \times 1{,}000 = 47{,}100$ $\quad$ $47{,}100 \div 1{,}000 = 47.1$

Pupils can use the Gattegno chart for support throughout this criterion, but by the end of year 6 they must be able to calculate without it.

Making connections

Writing multiples of powers of 10 depends on 6NPV–2. In 6AS/MD–2 pupils use their understanding of place-value and scaling number facts to manipulate equations.

6NPV–1 Example assessment questions

1. Complete the sentences.

   a. 500 made 1,000 times the size is ____________.

   b. 0.7 made 100 times the size is ____________.

   c. 800,000 made 10 times the size is ____________.

   d. 4,000,000 made one-thousandth times the size is ____________.

   e. 9,000 made one-hundredth times the size is ____________.

   f. 3 made one-tenth times the size is ____________.

2. The distance from London to Bristol is about 170km. The distance from London to Sydney, Australia is about 100 times as far. Approximately how far is it from London to Sydney?

3. A newborn elephant weighs about 150kg. A newborn kitten weighs about 150g. How many times the mass of a newborn kitten is a newborn elephant?

4. Walid has a place-value chart and three counters. He has represented the number 1,110,000.

Millions			Thousands			Ones		
100s	10s	1s	100s	10s	1s	100s	10s	1s
		●	●	●				

   a. Find 2 different numbers that Walid could make so that 1 number is one-hundredth times the size of the other number.

   b. Find 2 different numbers that Walid could make so that 1 number is 1,000 times the size of the other number.

5. Fill in the missing numbers.

×10 →		×10 →	
4.3	[ ]	[ ]	27,158
← ÷10		← ÷10	

×100 →

729 [ ] ← ÷100

×100 →

[ ] 5,806 ← ÷100

×1,000 →

14.3 [ ] ← ÷1,000

×1,000 →

[ ] 2,670,000 ← ÷1,000

6. Use the following to complete the equations:

×10 ×100 ×1,000 ÷10 ÷100 ÷1,000

Use each term only once.

543 [ ] = 5.43	3,169 [ ] = 3,169,000	515 [ ] = 5,150
276,104 [ ] = 27,610.4	35,000 [ ] = 35	427 [ ] = 42,700

6NPV–2 Place value in numbers up to 10,000,000

Recognise the place value of each digit in numbers up to 10 million, including decimal fractions, and compose and decompose numbers up to 10 million using standard and non-standard partitioning.

6NPV–2 Teaching guidance

Pupils must be able to read and write numbers up to 10,000,000, including decimal fractions. Pupils should be able to use a separator (such as a comma) every third digit from the decimal separator to help read and write numbers. Pupils must be able to copy numbers from calculator displays, inserting thousands separators and decimal points correctly. This will prepare them for secondary school, where pupils will be expected to know how to use calculators.

Pupils need to be able to identify the place value of each digit in a number.

Language focus

"In 67,000.4…

- the 6 represents 6 ten-thousands; the value of the 6 is 60,000
- the 7 represents 7 thousands; the value of the 7 is 7,000
- the 4 represents 4 tenths; the value of the 4 is 0.4"

Pupils must be able to combine units from millions to hundredths to compose numbers, and partition numbers into these units, and solve related addition and subtraction calculations. Pupils need to experience variation in the order of presentation of the units, so that they understand, for example, that 5,034,000.2 is equal to 4,000 + 30,000 + 0.2 + 5,000,000. Pupils should be able to represent a given number in different ways, including using place-value counters and Gattegno charts, and write numbers shown using these representations.

Pupils should then have sufficient understanding of the composition of large numbers to compare and order them by size.

Pupils also need to be able to solve problems relating to subtraction of any single place-value part from a number, for example:

381,920 – 900 = ☐

381,920 – ☐ = 380,920

As well as being able to partition numbers in the 'standard' way (into individual place-value units), pupils must also be able to partition numbers in 'non-standard' ways, and carry out related addition and subtraction calculations, for example:

518.32 + 30 = 548.32

381,920 – 60,000 = 321,920

Pupils can initially use place-value counters for support with this type of partitioning and calculation, but by the end of year 6 must be able to partition and calculate without them.

6NPV–2 Example assessment questions

1. What is the value of the digit 5 in each of these numbers?

   a. 720,541

   b. 5,876,023

   c. 1,587,900

   d. 651,920

   e. 905,389

   f. 2,120,806.50

   g. 8,002,345

   h. 701,003.15

2. Write a seven-digit number that includes the digit 8 once, where the digit has a value of:

   a. 8 million

   b. 8 thousand

   c. 8 hundred

   d. 80 thousand

3. Fill in the missing symbols (< or >).

   7,142,294 ◯ 7,124,294

   99,000 ◯ 600,000

   6,090,100 ◯ 690,100

   1,300,610 ◯ 140,017

   589,940 ◯ 1,010,222

4. Put these numbers in order from smallest to largest.

   8,102,304   8,021,403   843,021   8,043,021

6NPV–3 Numbers up to 10 million in the linear number system

Reason about the location of any number up to 10 million, including decimal fractions, in the linear number system, and round numbers, as appropriate, including in contexts.

6NPV–3 Teaching guidance

Pupils have already learnt about the location of whole numbers with up to 4 digits in the linear number system (1NPV–2, 2NPV–2, 3NPV–3 and 4NPV–3) and about the location of decimal fractions with up to 2 decimal places between whole numbers in the linear number system (5NPV–3). Pupils must now extend their understanding to larger numbers.

Pupils need to be able to identify or place numbers with up to 7 digits on marked number lines with a variety of scales, for example placing 12,500 on a 12,000 to 13,000 number line, and on a 10,000 to 20,000 number line.

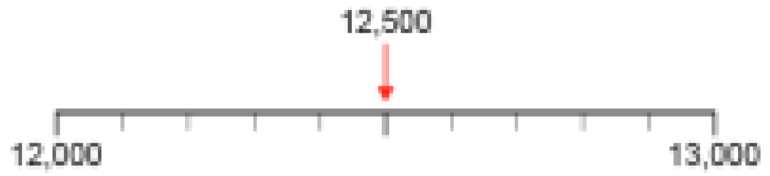


Figure 208: placing 12,500 on a 12,000 to 13,000 number line marked, but not labelled, in multiples of 100

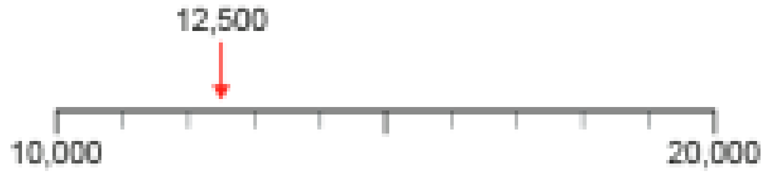


Figure 209: placing 12,500 on a 10,000 to 20,000 number line marked, but not labelled, in multiples of 1,000

Pupils need to be able to estimate the value or position of numbers on unmarked or partially marked numbers lines, using appropriate proportional reasoning.

Figure 210: estimating the position of 65,000 on an unmarked 50,000 to 100,00 number line

In the example below, pupils should reason: "a must be about 875,000 because it is about halfway between the midpoint of the number line, which is 850,000, and 900,000."

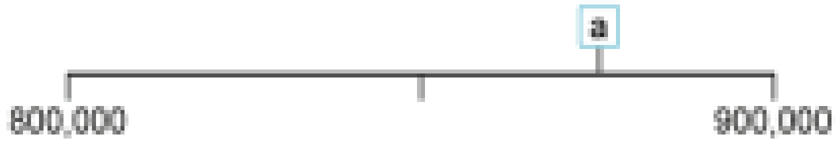


Figure 211: identifying 875,000 on a 800,00 to 900,000 number line marked only with a midpoint

Pupils should understand that, to estimate the position of a number with more significant digits on a large-value number line, they must attend to the leading digits and can ignore values in the smaller place-value positions. For example, when estimating the position of 5,192,012 on a 5,100,000 to 5,200,000 number line they only need to attend to the first 4 digits.

Pupils must also be able to round numbers in preparation for key stage 3, when they will learn to round numbers to a given number of significant figures or decimal places. They have already learnt to round numbers with up to 4 digits to the nearest multiple of 1,000, 100 and 10, and to round decimal fractions to the nearest whole number or multiple of 0.1. Now pupils should extend this to larger numbers. They must also learn that numbers are rounded for the purpose of eliminating an unnecessary level of detail. They must understand that rounding is a method of approximating, and that rounded numbers can be used to give estimated values including estimated answers to calculations.

Pupils should only be asked to round numbers to a useful and appropriate level: for example, rounding 7-digit numbers to the nearest 1 million or 100,000, and 6-digit numbers to the nearest 100,000 or 10,000. Pupils may use a number line for support, but by the end of year 6, they need to be able to round numbers without a number line. As with previous year groups (3NPV–3 , 4NPV–3 and 5NPV–3), pupils should first learn to identify the previous and next given multiple of a power of 10, before identifying the closest of these values. In the examples below, for 5,192,012, pupils must be able to identify the previous and next multiples of 1 million and 100,000, and round to the nearest of each.

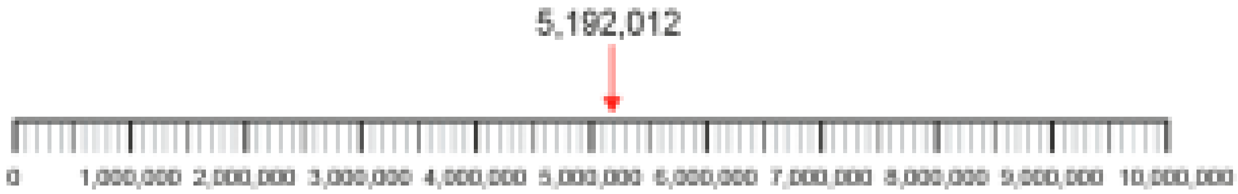


Figure 212: using a number line to identify the previous and next multiple of 1 million

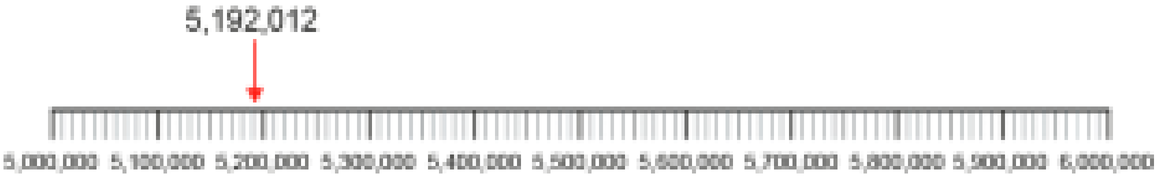


Figure 213: using a number line to identify the previous and next multiple of 100,000

> **Language focus**
>
> "The previous multiple of 1 million is 5 million. The next multiple of 1 million is 6 million."
>
> "The previous multiple of 100,000 is 5,100,000. The next multiple of 100,000 is 5,200,000."

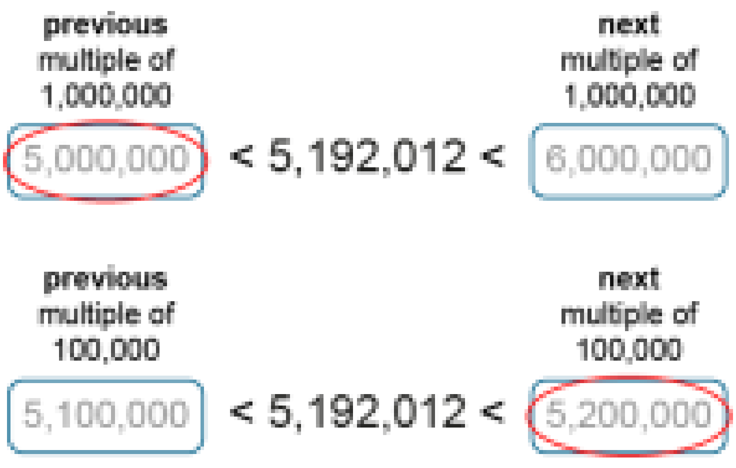


Figure 214: identifying the nearest multiple of 1 million and the nearest multiple of 100,000

> **Language focus**
>
> "The closest multiple of 1 million is 5 million."
> "5,192,012 rounded to the nearest million is 5 million."
>
> "The closest multiple of 100,000 is 5,200,000."
> "5,192,012 rounded to the nearest 100,000 is 5,200,000."

Pupils should explore the different reasons for rounding numbers in a variety of contexts, such as the use of approximate values in headlines, and using rounded values for estimates. They should discuss why a headline, for example, might use a rounded value, and when precise figures are needed.

Finally, pupils should also be able to count forwards and backwards, and complete number sequences, in steps of powers of 10 (1, 10, 100, 1,000, 10,000 and 100,000). Pay particular attention to counting over 'boundaries', for example:

- 2,100,000 2,000,000 1,900,000
- 378,500 379,500 380,500

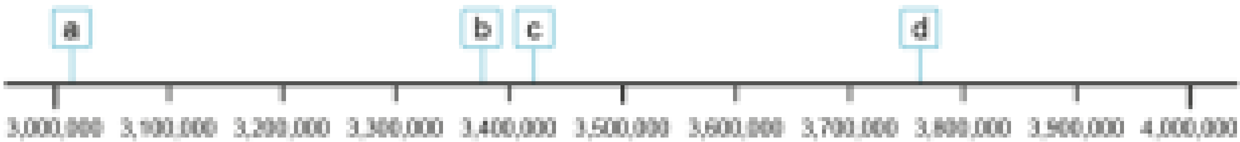
> **Making connections**
>
> Here, pupils must apply their knowledge from 6NPV–1, that each place value unit is made up of 10 of the unit to its right, to understand how each interval on a number line or scale is made up of 10 equal parts. This also links to 6NPV–4, in which pupils need to be able to read scales divided into 2, 4, 5 and 10 equal parts.

6NPV–3 Example assessment questions

1. Show roughly where each of these numbers is located on the number line below.

2,783,450 7,000,500 5,250,000 8,192,092 99,000

2. Estimate the values of a, b, c and d.



3. For each number:

- write the previous and next multiple of 1 million
- circle the previous or next multiple of 1 million which is closest to the number

previous multiple of 1,000,000		next multiple of 1,000,000
	< 2,783,450 <	
	< 5,192,012 <	
	< 5,811,159 <	
	< 7,683,102 <	

4. Fill in the missing numbers.

6,361,040	6,371,040	6,381,040	6,391,040	6,401,040	6,411,040		

2,004,567	2,003,567	2,002,567	2,001,567	2,000,567	1,999,567		

7,730,004		7,930,004	8,030,004		8,230,004		8,430,004

	9,149,301		9,129,301	9,119,301			9,089,301

5. What might the missing number be in this web page?

6. A swimming pool holds approximately 82,000 litres of water. The capacity of the swimming pool has been rounded to the nearest multiple of 1,000. Fill in the missing numbers to complete the sentences.

a. The minimum amount of water that the pool could hold is ____________.

b. The maximum amount of water that the pool could hold is ____________.

6NPV–4 Reading scales with 2, 4, 5 or 10 intervals

Divide powers of 10, from 1 hundredth to 10 million, into 2, 4, 5 and 10 equal parts, and read scales/number lines with labelled intervals divided into 2, 4, 5 and 10 equal parts.

6NPV–4 Teaching guidance

It is important for pupils to be able to divide powers of 10 into 2, 4, 5 or 10 equal parts because these are the intervals commonly found on measuring instruments and graph scales. Pupils have already learnt to divide 1, 100 and 1,000 in this way (5NPV–4, 3NPV–4 and 4NPV–4 respectively), and must now extend this to larger powers of 10. Pupils should be able to make connections between powers of 10, for example, describing similarities and differences between the values of the parts when 1 million, 1,000 and 1 are divided into 4 equal parts.

1,000,000			
250,000	250,000	250,000	250,000

1,000			
250	250	250	250

1			
0.25	0.25	0.25	0.25

Figure 215: bar models showing 1 million, 1,000 and 1 partitioned into 4 equal parts

Pupils should be able to skip count in these intervals forwards and backwards from any starting number (for example, counting forward from 800,000 in steps of 20,000, or counting backwards from 5 in steps of 0.25). This builds on counting in steps of 10, 20, 25 and 50 in year 3 (**3NPV–4**), in steps of 100, 200, 250 and 500 in year 4 (**4NPV–4**), and in steps of 0.1, 0.2, 0.25 and 0.5 in year 5 (**5NPV–4**).

Pupils should practise reading measurement and graphing scales with labelled power-of-10 intervals divided into 2, 4, 5 and 10 equal parts.

Pupils need to be able to write and solve addition, subtraction, multiplication and division equations related to powers of 10 divided into 2, 4, 5 and 10 equal parts, as exemplified for 1 million and 4 equal parts below. Pupils should be able to connect finding equal parts of a power of 10 to finding $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{5}$ or $\frac{1}{10}$ of the value.

$750{,}000 + 250{,}000 = 1{,}000{,}000$

$1{,}000{,}000 - 250{,}000 = 750{,}000$

$1{,}000{,}000 - 750{,}000 = 250{,}000$

$1{,}000{,}000 \div 4 = 250{,}000$

$1{,}000{,}000 \div 250{,}000 = 4$

$4 \times 250{,}000 = 1{,}000{,}000$

$250{,}000 \times 4 = 1{,}000{,}000$

$\frac{1}{4}$ of $1{,}000{,}000 = 250{,}000$

Making connections

Dividing powers of 10 into 10 equal parts is also assessed as part of 6NPV–1.

Reading scales also builds on number-line knowledge from 6NPV–3. Conversely, experience of working with scales with 2, 4, 5 or 10 divisions in this criterion improves pupils' estimating skills when working with unmarked number lines and scales as described in **6NPV–3**.

6NPV–4 Example assessment questions

1. If $\frac{1}{10}$ of a 1kg bag of flour is used, how much is left?

2. In 2005, the population of Birmingham was about 1 million. At that time, about $\frac{1}{5}$ of the population was over 60 years old. Approximately how many over-60s lived in Birmingham in 2005?

3. A builder ordered 1,000kg of sand. She has about 300kg left. What fraction of the total amount is left?

4. Fill in the missing parts.

10,000,000			

1,000									

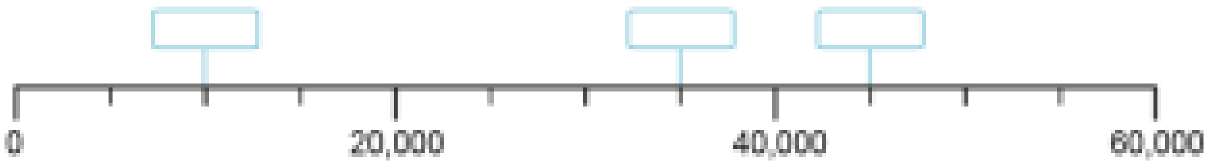
250,000	250,000	250,000	250,000

1			

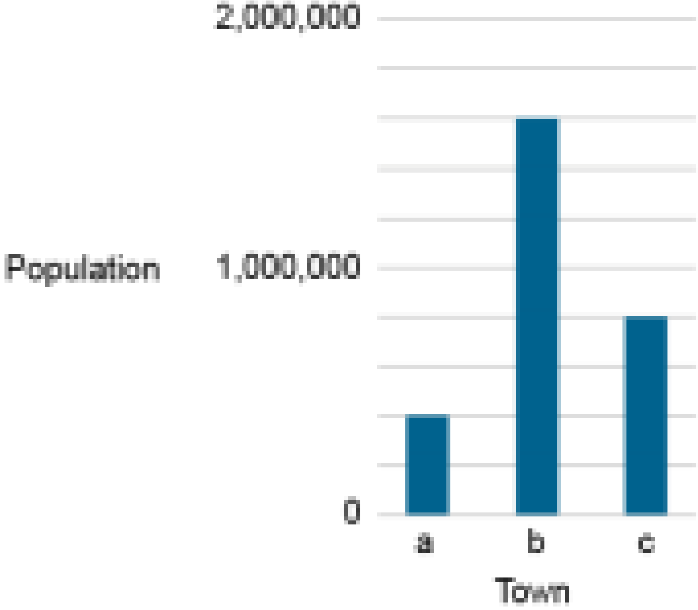
1,000,000				

0.5	0.5

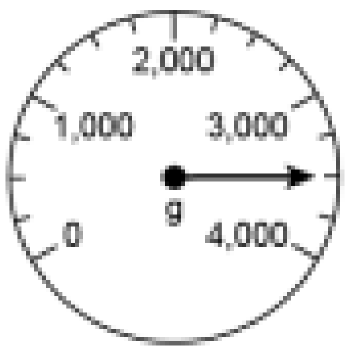
5. Fill in the missing numbers.



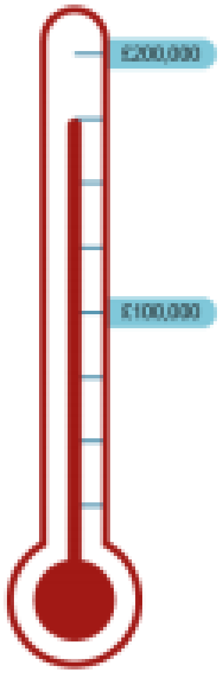
6. The bar chart shows the approximate populations of 3 different towns. What are the populations?



7. What mass does each scale show?



8. Some children are trying to raise £200,000 for charity. The diagram shows how much they have raised so far.



a. How much money have they raised?

b. How much more money do they need to raise to meet their target?

6AS/MD–1 Quantify additive and multiplicative relationships

Understand that 2 numbers can be related additively or multiplicatively, and quantify additive and multiplicative relationships (multiplicative relationships restricted to multiplication by a whole number).

6AS/MD–1 Teaching guidance

Throughout key stage 2, pupils have learnt about and used 2 types of mathematical relationship between numbers: additive relationships and multiplicative relationships. In year 6, pupils should learn to represent the relationship between 2 given numbers additively or multiplicatively, as well as use such a representation to calculate a missing number, including in measures and statistics contexts.

Consider the following: Holly has cycled 20km. Lola has cycled 60km.

We can describe the relationship between the distances either additively (Lola has cycled 40km further than Holly; Holly has cycled 40km fewer than Lola) or multiplicatively

(Lola has cycled 3 times the distance that Holly has cycled). The relationship between the numbers 20 and 60 can be summarised as follows.

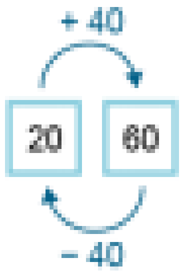


Figure 216: additive relationship between 20 and 60

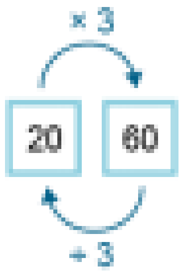


Figure 217: multiplicative relationship between 20 and 60

> **Language focus**
>
> "The relationship between 2 numbers can be expressed additively or multiplicatively."

As pupils progress into key stage 3, the ability to relate, recognise and use multiplicative relationships is essential. A pupil who can think multiplicatively would, for example, calculate the cost of 1.2m of ribbon at 75p per metre as $1.2 \times 75\text{p}$, whereas a pupil who was still thinking only in terms of additive relationships would use the approach of finding the cost of 0.2m (15p) and adding it to the cost of 1m (75p). During key stage 3, pupils will regularly use calculators to solve problems with this type of structure, and the multiplicative approach is more efficient because it involves fewer steps.

Given any 2 numbers (related by a whole-number multiplier), pupils must be able to identify the additive relationship (in the example above, $+40$ and -40) and the multiplicative relationship (in the example above $\times 3$ and $\div 3$). Though multiplicative relationships should be restricted to whole-number multipliers, pupils should be able to connect division by the whole number to scaling by a unit fraction: in the example above, this corresponds to understanding that because $60 \div 3 = 20$, 20 is one-third times the size of 60.

When given a sequence of numbers, pupils should be able to identify whether the terms are all related additively or multiplicatively, identify the specific difference or multiplier and use this to continue a sequence either forwards or backwards. Pupils will need to use formal written methods to calculate larger numbers in sequences.

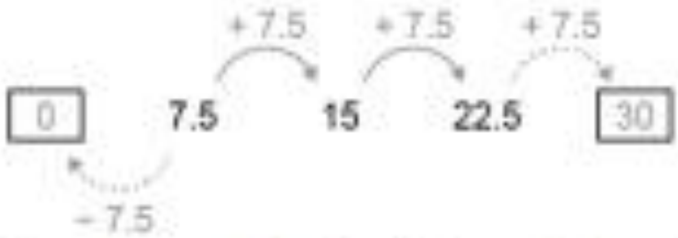


Figure 218: completing a sequence where the difference between adjacent terms is 7.5

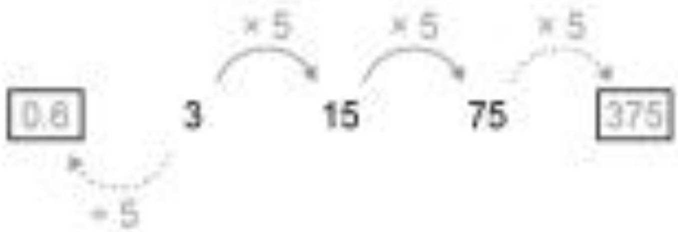


Figure 219: completing a sequence where each term is 5 times the previous

Making connections

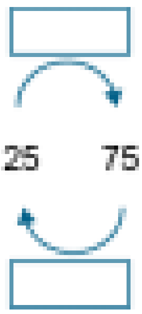
In 6AS/MD–4 pupils solve problems with 2 unknowns, where the relationship between the unknowns may be additive, multiplicative or both, for example: find 2 numbers, where one is 3 times the size of the other, and the difference between them is 40.

6AS/MD–1 Example assessment questions

1. Fill in the missing numbers.

300 + □ = 1,200	75 = 3 + □	□ + 0.1 = 10
300 × □ = 1,200	75 = 3 × □	□ × 0.1 = 10

2. Write an expression in each box to show the relationship between numbers 25 and 75. Is there more than one way to answer this question? Explain.



3. The examples below show the first 2 numbers in a sequence. Find 2 different ways to continue each sequence, using addition for the first and multiplication for the second.

a.	4	16		or	4	16	
b.	2	200		or	2	200	
c.	0.01	10		or	0.01	10	

4. Complete these sequences.

0.5	5	9.5				27.5	32

	0.5	0.75	1		
	25	125	625		
0.2	6	180			

6AS/MD–2 Derive related calculations

Use a given additive or multiplicative calculation to derive or complete a related calculation, using arithmetic properties, inverse relationships, and place-value understanding.

6AS/MD–2 Teaching guidance

In previous year groups in key stage 2 pupils have learnt about and used the commutative and associative properties of addition (3AS–3), and the commutative, associative and distributive properties of multiplication (4MD–2 and 4MD–3).

Pupils have also implicitly used the compensation property of addition, for example, when partitioning two-digit numbers in different ways in year 2:

$70 + 2 = 72$ $60 + 12 = 72$

In year 6, pupils should learn the compensation property of addition.

> **Language focus**
>
> "If one addend is increased and the other is decreased by the same amount, the sum stays the same."

Pupils should be able to use the compensation property of addition to complete equations such as $25 + 35 = 27.5 + ?$, and to help them solve calculations such as 27.5 + 32.5.

Similarly, pupils may have implicitly used the compensation property of multiplication, for example, when recognising connections between multiplication table facts:
$5 \times 8 = 10 \times 4$

In year 6, pupils should learn the compensation property of multiplication.

> **Language focus**
>
> "If I multiply one factor by a number, I must divide the other factor by the same number for the product to stay the same."

Pupils should be able to use the compensation property of multiplication to complete equations such as $0.3 \times 320 = 3 \times ?$, and to help them solve calculations such as 0.3×320.

Pupils have extensive experience about the effect on the product of scaling one factor from 3NF–3, 4F–3 and 5NF–2, where they learnt to scale known number facts by 10, 100, one-tenth and one-hundredth. Now they can generalise.

Language focus

"If I multiply one factor by a number, and keep the other factor the same, I must multiply the product by the same number."

Pupils should practise combining their knowledge of arithmetic properties and relationships to solve problems such as the examples here and in the Example assessment questions below.

Example problem 1

Question: Explain how you would use the first equation to complete the second equation:

$2,448 \div 34 = 72$

$72 \times \square = 24,480$

Explanation:

1. Use the inverse relationship between multiplication and division to restate the equation:
   $72 \times 34 = 2,448$
2. Apply understanding of place-value: the product can be made 10 times the size by making one of the factors 10 times the size.
   $72 \times 340 = 24,480$

Example problem 2

Question: Explain how you would use the first equation to complete the second equation:

$921 = 349 + 572$

$92.1 = 44.9 + \square$

Explanation:

1. Apply understanding of place value, making the sum and addends 1 tenth times the size.
   $92.1 = 34.9 + 57.2$
2. Apply the compensation property of addition to solve the equation: add 10 to the first addend and subtract 10 from the second addend.
   $92.1 = 44.9 + 47.2$

Pupils should learn to write a series of written equations to justify their solutions.

Being able to work fluently with related equations in this way will prepare pupils for manipulating algebraic equations in key stage 3 and writing proofs.

Pupils can already apply place-value understanding to known multiplication facts to scale one factor, for example, $3 \times 4 = 12$, so $3 \times 40 = 120$. Now they should extend this to scaling both factors, for example, $3 \times 4 = 12$, so $30 \times 40 = 1{,}200$.

> **Making connections**
>
> In this criterion, pupils use their understanding from 6NPV–1 of scaling numbers by 10, 100 and 1,000.

6AS/MD–2 Example assessment questions

1. Fill in the missing numbers.

   $327 + 278 = 330 + \square$ $25 \times 48 = 50 \times \square$

2. $327 + 515 = 842$
   Use this calculation to complete the following equations.

   $\square + 51.5 = 84.2$

   $8{,}420 - \square = 3{,}270$

   $84{,}200 - 51{,}500 = \square$

3. $21{,}760 = 256 \times 85$
   Use this calculation to complete the following equations.

   $256 \times 8.5 = \square$

   $2{,}560 \times 85 = \square$

   $2{,}176 \div 85 = \square$

4. $3{,}128 \div 23 = 136$

   Use the division calculation so solve the following calculation. Explain your answer.

   $24 \times 136 = \square$

5. Fill in the missing number.

   $25 \times 60 = \square \times 60 + 120$

6AS/MD–3 Solve problems involving ratio relationships

Solve problems involving ratio relationships.

6ASMD–3 Teaching guidance

Pupils already have the arithmetic skills to solve problems involving ratio. They should now learn to describe 1-to-many (and many-to-1) correspondence structures.

Language focus

"For every 1 cup of rice you cook, you need 2 cups of water."

"For every 10 children on the school trip, there must be 1 adult."

Pupils should learn to complete ratio tables, given a 1-to-many or many-to-1 relationship.

cups of rice	1	2	3	4	5	6
cups of water	2	4	6	8	10	12

number of children	10	20	30	40	50	60
number of adults	1	2	3	4	5	6

Pupils must recognise that proportionality is preserved in these contexts, for example, there is always twice the volume of water needed compared to the volume or rice, regardless of how much rice there is. This will prepare pupils for key stage 3, when they will learn to describe correspondence structures using ratio notation and to express ratios in their simplest forms.

Pupils should be able to recognise a 1-to-many or many-to-1 structure, without it being explicitly given and use the relationship to solve problems. For example, here pupils should recognise that, in both examples, for every 1 red bead there are 3 blue beads (or for every 3 blue beads there is 1 red bead), irrespective of the arrangement of the beads.

Figure 220: bead strings, each with the structure 'for every 1 red bead, there are 3 blue beads'

For examples like this, pupils should also be able to include the total quantity in a table.

number of red beads	1	2	3	4
number of blue beads	3	6	9	12
total number of beads	**4**	**8**	**12**	**16**

Pupils should also be able to answer questions such as:

- if there were 5 red beads, how many blue beads would there be?
- if there were 21 blue beads, how many beads would there be altogether?
- if there were 40 beads altogether, how many red beads and how many blue beads would there be?

Pupils must also learn to describe and solve problems related to many-to-many structures.

Language focus

"For every 2 yellow beads there are 3 green beads".

Pupils may initially use manipulatives, such as cubes or beads, for support, but by the end of year 6, they must be able to complete many-to-many correspondence tables and solve related problems without manipulatives.

number of yellow beads	2	4	6	8
number of green beads	3	6	9	12
total number of beads	**5**	**10**	**15**	**20**

Pupils should also begin to prepare for using the unitary method at key stage 3, when it is required for unit conversions, percentage calculations and other multiplicative problems. For example, if they are given a smoothie recipe for 2 people (20 strawberries, 1 banana and 150ml milk), they should be able to adjust the recipe by multiplying or dividing by a whole number, for example, dividing the quantities by 2 to find the amounts for 1 person, or multiplying the quantities by 3 to find the amounts for 6 people. At key stage 3, pupils would then, for example, be able to use the unitary method to adjust the recipe for 5 people, via calculating the amounts for 1 person.

Making connections

To recognise a one-to-many or many-to-one structure, pupils need to be able to identify multiplicative relationships between given numbers (6AS/MD–2).

6AS/MD–3 Example assessment questions

1. For every 1 litre of petrol, Miss Smith's car can travel about 7km.
   a. How many kilometres can Miss Smith's car travel on 6 litres of petrol?
   b. Miss Smith lives about 28km from school. How many litres of petrol does she use to get to school?
2. For every 3m of fence I need 4 fence panels. The fence will be 15m long. How many fence panels will I need?
3. I am decorating a cake with fruit. I use 2 raspberries for every 3 strawberries. Altogether I put 30 berries on the cake.
   a. How many raspberries did I use?
   b. How many strawberries did I use?
4. For every 500g of excess baggage I take on an aeroplane, I must pay £7.50. I have 3.5kg of excess baggage. How much must I pay?

5. Lily and Ralph are eating grapes. The diagram represents the relationship between the number of grapes that the children eat.

Fill in the missing numbers.

Number of grapes that Lily eats	Number of grapes that Ralph eats
1	
	20
3	

6. Giya is planting flowers in her garden. For every 5 red flowers she plants, she plants 3 yellow flowers. If Giya plants 18 yellow flowers, how many red flowers does she plant?

7. I am making a necklace. So far, it has 4 black beads and 1 white bead. How many more white beads would I need to add so that there are 4 white beads for every 1 black bead?

6AS/MD–4 Solve problems with 2 unknowns

Solve problems with 2 unknowns.

6AS/MD–4 Teaching guidance

Pupils need to be able to solve problems with 2 unknowns where:

- there are an infinite number of solutions
- there is more than 1 solution
- there is only 1 solution

Pupils may have seen equations with 2 unknowns before, for example, when recognising connections between multiplication table facts:

$5 \times \square = 10 \times \square$

In year 6, pupils must recognise that an equation like this has many (an infinite number) of solutions. They should learn to provide example solutions by choosing a value for one unknown and then calculating the other unknown.

Pupils should be able to solve similar problems where there is more than one solution, but not an infinite number, for example:

Danny has some 50p coins and some 20p coins. He has £1.70 altogether. How many of each type of coin might he have?

In these cases, pupils may choose a value for the first unknown and be unable to solve the equation for the other unknown (pupils may first set the number of 50p pieces at 2, giving £1, only to find that it is impossible to make up the remaining 70p from 20p coins). Pupils should then try a different value until they find a solution. For a bound problem with only a few solutions, like the coin example, pupils should be able to find all possible solutions by working systematically using a table like that shown below. They should be able to reason about the maximum value in each column.

Number of coins	Quantity in 50p coins	Quantity in 20p coins
1	50p	20p
2	£1	40p
3	£1.50	60p
4		80p
5		£1
6		£1.20
7		£1.40
8		£1.60

Figure 221: finding the 2 solutions to the coin problem: one 50p coin and six 20p coins, or three 50p coins and one 20p coin

Pupils must also learn to solve problems with 2 unknowns that have only 1 solution. Common problems of this type involve 2 pieces of information being given about the relationship between the 2 unknowns – 1 piece of additive information and either another piece of additive information or a piece of multiplicative information. Pupils should learn to draw models to help them solve this type of problem.

Example problem 1

Question: The sum of 2 numbers is 25, and the difference between them is 7. What are the 2 numbers?

Solution:

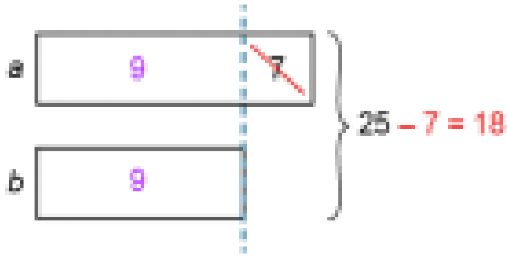


Figure 222: using a bar model to solve a problem with 2 unknowns – example 1

$a = 9 + 7 = 16$
$b = 9$

The numbers are 16 and 9.

Example problem 2

Question: The sum of 2 numbers is 48. One number is one-fifth times the size of the other number. What are the 2 numbers?

Solution:

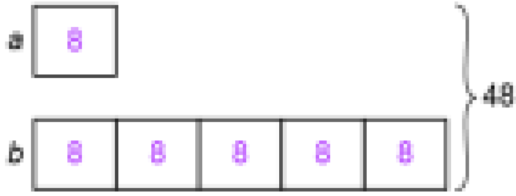
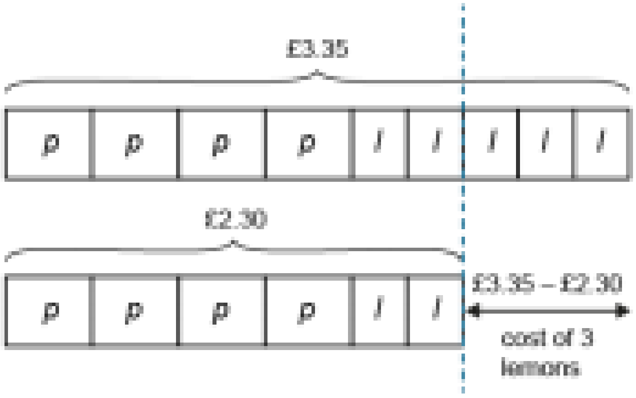


Figure 223: using a bar model to solve a problem with 2 unknowns – example 2

$a = 8$
$b = 5 \times 8 = 40$

The numbers are 8 and 40.

Pupils should also be able to use bar modelling to solve more complex problems with 2 unknowns and 1 solution, such as: 4 pears and 5 lemons cost £3.35. 4 pears and 2 lemons cost £2.30. What is the cost of 1 lemon?



cost of 3 lemons = £3.35 – £2.30 = £1.05

so

cost of 1 lemon = £1.05 ÷ 3 = £0.35

Figure 224: using a bar model to solve a problem with 2 unknowns – example 3

Solving problems with 2 unknowns and 1 solution will prepare pupils for solving simultaneous equations in key stage 3.

Pupils should practise solving a range of problems with 2 unknowns, including contextual measures and geometry problems.

Making connections

Within this criterion, pupils must be able to use their understanding of how 2 numbers can be related additively or multiplicatively (6AS/MD–1). In 6G–1 pupils solve geometry problems with 2 unknowns, for example, finding the unknown length and unknown width of a rectangle with a perimeter of 14cm.

6AS/MD–4 Example assessment questions

1. A baker is packing 60 cakes into boxes. A small box can hold 8 cakes and a large box can hold 12 cakes.

   a. How many different ways can he pack the cakes?

   b. How can he pack the cakes with the fewest number of boxes?

2. 1 eraser and 5 pencils cost a total of £3.35.

   5 erasers and 5 pencils cost a total of £4.75.

   a. How much does 1 eraser cost?

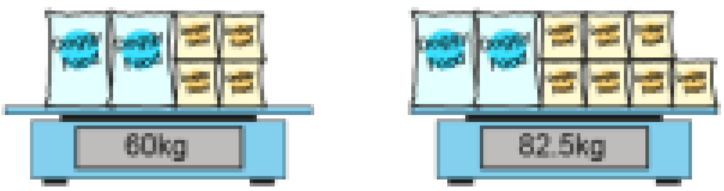
   b. How much does 1 pencil cost?

3. An adult ticket for the zoo costs £2 more than a child ticket. I spend a total of £33 buying 3 adult and 2 child tickets.

   a. How much does an adult ticket cost?

   b. How much does a child ticket cost?

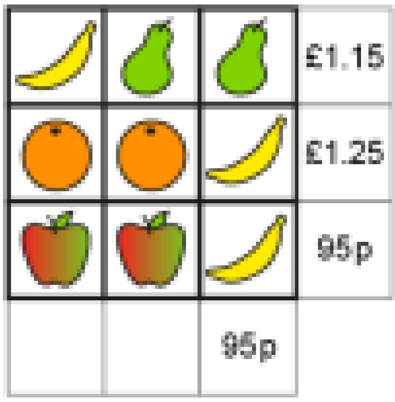
4. The balances show the combined masses of some large bags of dog food and some small bags of dog food.



   How much does each bag-size cost?

5. A rectangle with side-lengths *a* and *b* has a perimeter of 30cm. *a* is a 2-digit whole number and *b* is a 1-digit whole number. What are the possible values of *a* and *b*?

6. The diagram shows the total cost of the items in each row and column. Fill in the 2 missing costs.



6F–1 Simplify fractions

Recognise when fractions can be simplified, and use common factors to simplify fractions.

6F–1 Teaching guidance

In year 5, pupils learnt to find equivalent fractions (5F–2). Now pupils must build on this and learn to recognise when fractions are not in their simplest form. They should use their understanding of common factors (5MD–2) to simplify fractions.

Pupils should learn that when the numerator and denominator of a fraction have no common factors (other than 1) then the fraction is in its simplest form. Pupils should learn that a fraction can be simplified by dividing both the numerator and denominator by a common factor. They must realise that simplifying a fraction does not change its value, and the simplified fraction has the same position in the linear number system as the original fraction. Pupils should begin with fractions where the numerator and denominator have only one common factor other than 1, for example $\frac{6}{15}$.

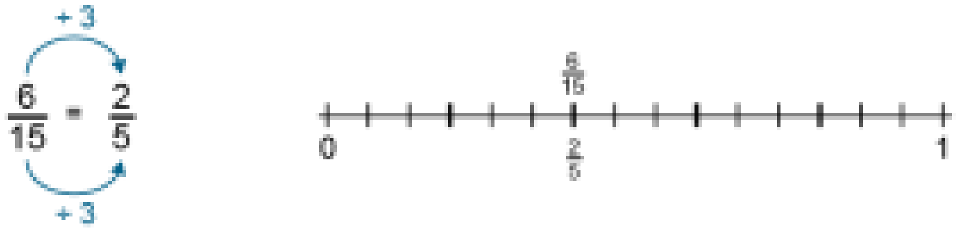


Figure 225: simplifying $\frac{6}{15}$ by dividing the numerator and denominator by the common factor of 3

> **Language focus**
>
> "A fraction can be simplified when the numerator and denominator have a common factor other than 1."

Pupils should then learn to simplify fractions where the numerator and denominator share several common factors, for example $\frac{4}{12}$. Pupils should understand that they should divide the numerator and denominator by the highest common factor to express a fraction in its simplest form, but that the simplification can also be performed in more than 1 step.

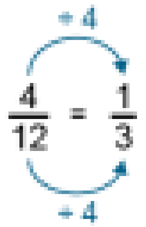


Figure 226: simplifying $\frac{4}{12}$ by dividing the numerator and denominator by the highest common factor

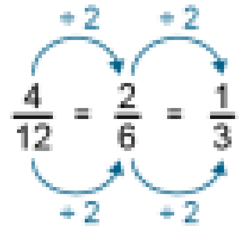


Figure 227: simplifying $\frac{4}{12}$ in 2 steps

> **Language focus**
>
> "To convert a fraction to its simplest form, divide both the numerator and the denominator by their highest common factor."

Pupils should learn to always check their answer when simplifying a fraction to confirm that it is in its simplest form and the only remaining common factor is 1.

Pupils should be able to simplify fractions:

- where the numerator is a factor of the denominator (and therefore also the highest common factor), for example, $\frac{3}{9}$ or $\frac{7}{28}$, resulting in a simplified fraction that is a unit fraction
- where the numerator is not a factor of the denominator, for example, $\frac{4}{14}$ or $\frac{15}{20}$, resulting in a simplified fraction that is a non-unit fraction

In year 4 pupils learnt to convert between mixed numbers and improper fractions (4F–2) and to add and subtract fractions to give a sum greater than 1 (4F–3). This criterion on simplifying fractions provides an opportunity for pupils to continue to practise these skills as they learn how to simplify fractions with a value greater than 1.

Pupils should consider calculations such as $\frac{9}{12} + \frac{11}{12}$ and understand that the resulting improper fraction, $\frac{20}{12}$, can be simplified either directly, or by first converting to a mixed number and then simplifying the fractional part.

$$\frac{20}{12} \overset{\div 4}{=} \frac{5}{3} = 1\frac{2}{3}$$

Figure 228: simplifying $\frac{20}{12}$ to $\frac{5}{3}$, then converting to a mixed number

$$\frac{20}{12} = 1\frac{8}{12} \overset{\div 4}{=} 1\frac{2}{3}$$

Figure 229: converting $\frac{20}{12}$ to $\frac{5}{3}$, then simplifying

6F–1 Example assessment questions

1. Sort these fractions according to whether they are expressed in their simplest form or not.

$\frac{3}{15}$ $\frac{2}{5}$ $\frac{4}{20}$ $\frac{25}{38}$ $\frac{1}{8}$ $\frac{7}{21}$ $\frac{18}{30}$ $\frac{9}{17}$ $\frac{5}{15}$ $\frac{11}{20}$ $\frac{23}{30}$

Fraction in its simplest form	Fraction not in its simplest form

2. Solve these calculations, giving each answer in the simplest form.

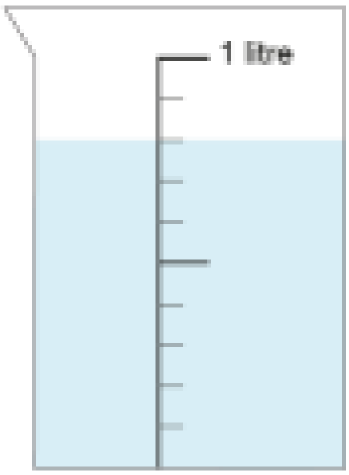
$\frac{2}{9}+\frac{4}{9}$	$\frac{3}{7}-\frac{1}{7}$	$\frac{4}{15}+\frac{2}{15}$
$\frac{5}{12}+\frac{5}{12}-\frac{2}{12}$	$\frac{2}{13}+\frac{7}{13}-\frac{4}{13}$	$\frac{4}{5}+\frac{4}{5}$
$\frac{7}{10}+\frac{5}{10}+\frac{3}{10}$	$\frac{8}{9}+\frac{8}{9}-\frac{1}{9}$	$3\frac{7}{10}+2\frac{9}{10}$
$\frac{13}{8}+\frac{11}{8}$	$7\frac{1}{6}-1\frac{2}{6}$	$\frac{17}{3}-\frac{5}{3}$

3. Ahmed says, "To simplify a fraction, you just halve the numerator and halve the denominator." Is Ahmed's statement always true, sometimes true or never true? Explain your answer.

4. Put these numbers in order from smallest to largest by simplifying them to unit fractions.

$\frac{3}{18}$ $\frac{5}{20}$ $\frac{4}{8}$ $\frac{2}{18}$ $\frac{4}{12}$ $\frac{6}{60}$

5. How much water is in this beaker? Write your answer as a fraction of a litre in its simplest form.



6F–2 Express fractions in a common denomination

Express fractions in a common denomination and use this to compare fractions that are similar in value.

6F–2 Teaching guidance

Pupils should already be able to identify multiples of numbers, and common multiples of numbers within the multiplication tables (4NF–1 and 5MD–2). To be ready to progress to key stage 3, given 2 fractions pupils must be able to express them with the same denominator.

Pupils should first work with pairs of fractions where one denominator is a multiple of the other, for example, $\frac{1}{5}$ and $\frac{4}{15}$. They should learn that the denominator that is the multiple (here 15) can be used as a common denominator. Pupils should then be able to apply what they already know about writing equivalent fractions (**5F–2**) to express the fractions in a common denomination.

Language focus

"We need to compare the denominators of $\frac{1}{5}$ and $\frac{4}{15}$."

"15 is a multiple of 5."

"We can use 15 as the common denominator."

"We need to express both fractions in fifteenths."

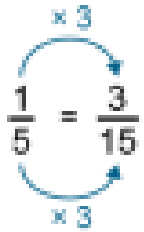


Figure 230: expressing $\frac{1}{5}$ in fifteenths

Pupils must then learn to work with pairs of fractions where one denominator is not a multiple of the other, for example, $\frac{1}{3}$ and $\frac{3}{8}$. Pupils should recognise when 1 denominator is not a multiple of the other (here, 3 is not a factor of 8 and 8 is not a multiple of 3) and learn to identify a new denominator that is a common multiple of both denominators. Again, pupils should then be able to apply what they already know about writing equivalent fractions to express the fractions in a common denomination.

Language focus

"We need to compare the denominators of $\frac{1}{3}$ and $\frac{3}{8}$."

"8 is not a multiple of 3."

"24 is a multiple of both 3 and 8."

"We can use 24 as the common denominator."

"We need to express both fractions in twenty-fourths."

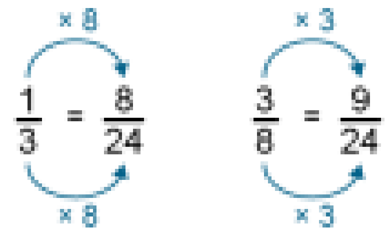


Figure 231: expressing $\frac{1}{3}$ and $\frac{3}{8}$ in twenty-fourths

At key stage 3, pupils will learn to find the lowest common multiple of any 2 numbers. At key stage 2, being able to find a common multiple of the denominators by multiplying the 2 denominators is sufficient.

Language focus

"If one denominator is not a multiple of the other, we can multiply the two denominators to find a common denominator."

This does not always result in the lowest common denominator (for example $\frac{1}{6}$ and $\frac{2}{9}$ can be expressed with a common denominator of 18 rather than 54), and if pupils can identify a lower common denominator then they should use it. Pupils may also recognise when the resulting pair of common denomination fractions can be simplified.

Pupils should also be able to find a common denominator for more than 2 fractions, such as $\frac{1}{3}$, $\frac{4}{15}$ and $\frac{2}{5}$, when 1 of the denominators is a multiple of the other denominators (in this case 15).

Once pupils know how to express fractions with a common denominator, they can use this to compare and order fractions.

> **Language focus**
>
> "If the denominators are the same, then the larger the numerator, the larger the fraction."

> **Making connections**
>
> In 6F-1 <link> pupils learnt to simplify fractions, and applied this to compare and order fractions that can be simplified to unit fractions (see 6F-1 Example assessment questions <link>, question 4). In this criterion, pupils learnt to express fractions in a common denomination, and use this to compare and order fractions. Pupils should be able to identify which method is appropriate for a given pair or set of fractions. Pupils learn more about comparing fractions, and choosing an appropriate method in 6F-3 <link>.

6F–2 Example assessment questions

1. Fill in the missing symbols (<, > or =). You will need to simplify some of the fractions and express each pair with a common denominator.

$\frac{5}{7} \bigcirc \frac{2}{3}$ $\frac{6}{10} \bigcirc \frac{3}{5}$

$\frac{7}{9} \bigcirc \frac{3}{4}$ $\frac{5}{7} \bigcirc \frac{6}{8}$

$\frac{2}{3} \bigcirc \frac{7}{10}$ $\frac{2}{6} \bigcirc \frac{3}{9}$

$\frac{3}{11} \bigcirc \frac{1}{3}$ $\frac{1}{5} \bigcirc \frac{2}{11}$

2. Express each set of fractions with a common denominator. Then put them in order from smallest to largest.

a. $\frac{4}{20}$ $\frac{1}{4}$ $\frac{3}{10}$ $\frac{2}{5}$

b. $\frac{2}{9}$ $\frac{1}{3}$ $\frac{1}{6}$ $\frac{4}{18}$

3. Ahmed has a beaker containing $\frac{7}{10}$ of a litre of water. Imran has a beaker containing $\frac{3}{5}$ of a litre of water. Express the fractions with a common denominator to work out whose beaker contains the most water.
4. Ben and Felicity are both trying to raise the same amount of money for charity. So far, Ben has raise $\frac{3}{4}$ of the amount, while Felicity has raised $\frac{5}{7}$ of the amount. Express the fractions with a common denominator to work out who is closest to meeting their target.

6F–3 Compare fractions with different denominators

Compare fractions with different denominators, including fractions greater than 1, using reasoning, and choose between reasoning and common denomination as a comparison strategy.

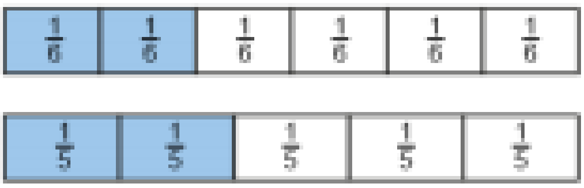
6F–3 Teaching guidance

In 6F–2 pupils learnt to compare any 2 fractions by expressing them with a common denominator. However fractions can often be compared by reasoning, without the need to express them with a common denominator.

Pupils can already compare unit fractions.

Language focus
"If the numerators are both 1, then the larger the denominator, the smaller the fraction."

Pupils should now extend this to compare other fractions with the same numerator, for example, because $\frac{1}{5}$ is greater than $\frac{1}{6}$ we know that $\frac{2}{5}$ is greater than $\frac{2}{6}$.



$$\frac{2}{5} > \frac{2}{6}$$

Figure 232: bar models to compare $\frac{2}{5}$ and $\frac{2}{6}$

Language focus

"If the numerators are the same, then the larger the denominator, the smaller the fraction."

Pupils should be able to use reasoning in other ways when comparing fractions:

- For each fraction they should be able to visualise where it is positioned on a number line, for example, thinking about whether it is greater than or less than $\frac{1}{2}$ or whether it is close to 0 or 1.
- Pupils should be able to reason about the relationship between the numerator and the denominator of each fraction, asking themselves 'Is this fraction a large or small part of the whole?'. They should be able to reason, for example, $\frac{5}{6}$ is greater than $\frac{7}{11}$ because 5 is a larger part of 6 than 7 is of 11.
- For fractions that are a large part of the whole, pupils should be able to reason about how close each fraction is to the whole. For example, $\frac{7}{8}$ is $\frac{1}{8}$ less than the whole, while $\frac{6}{7}$ is $\frac{1}{7}$ less than the whole. Since $\frac{1}{8}$ is less than than $\frac{1}{7}$, $\frac{7}{8}$ must be larger than $\frac{6}{7}$.

For a given pair or set of fractions, pupils must learn to assess whether it is more appropriate to compare them using reasoning or to express them in a common denomination.

Making connections

When given a pair or set of fractions to compare, pupils may need to convert some of the fractions to their simplest form (6F–1). They then need to assess whether it is more appropriate to compare them using reasoning (this criterion) or to express them in a common denomination (6F–2).

6F–3 Example assessment questions

1. Which number(s) could go in the missing-number box to make this statement true?

$\frac{1}{4} > \frac{1}{\square} > \frac{1}{10}$

2. Without using a common denominator, put each set of fractions in order from smallest to largest.

a. $\frac{10}{8}$ $\frac{7}{8}$ $\frac{5}{8}$ $\frac{3}{8}$ $\frac{8}{8}$ $\frac{4}{8}$ $\frac{2}{8}$

b. $\frac{1}{6}$ $\frac{1}{5}$ $\frac{1}{8}$ $\frac{1}{7}$ $\frac{1}{10}$ $\frac{1}{9}$

c. $\frac{3}{3}$ $\frac{3}{8}$ $\frac{3}{11}$ $\frac{3}{100}$ $\frac{3}{5}$ $\frac{3}{2}$

3. Sabijah and Will are in a running race. Sabijah has run $\frac{9}{10}$ of the race. Will has run $\frac{8}{9}$ of the race. Who is further ahead? Explain your reasoning.

4. Fill in the missing symbols (<, > or =).

$\frac{5}{6} \bigcirc \frac{4}{7}$ $\frac{8}{9} \bigcirc \frac{7}{11}$

5. Think of a number that can go in each box so that the fractions are arranged in order from smallest to largest.

$\frac{1}{3}$ $\frac{\square}{5}$ $\frac{1}{\square}$ $\frac{4}{7}$ $\frac{3}{\square}$

6G–1 Draw, compose and decompose shapes

Draw, compose, and decompose shapes according to given properties, including dimensions, angles and area, and solve related problems.

6G–1 Teaching guidance

Through key stage 2, pupils have learnt to measure perimeters, angles and areas of shapes, and have learnt to draw polygons by joining marked points (3G–2) and draw angles of a given size (5G–1). By the end of year 6, pupils must be able to draw, compose and decompose shapes defined by specific measurements. Composing and decomposing shapes prepares pupils for solving geometry problems at key stage 3, for example, finding the area of a trapezium by decomposing it to a rectangle and 2 triangles.

Pupils should be able to draw a named shape to meet a given measurement criterion, for example:

- drawing a rectangle, on squared-centimetre paper, with a perimeter of 14cm (Example 1 below)
- drawing a pentagon, on squared-centimetre paper, with an area of $10cm^2$ (Example 2 below)
- drawing a triangle at actual size, based on a sketch with labelled lengths and angles (see 6G–1, question 5)

Example 1

Task: draw a rectangle with a perimeter of 14cm.

Example solution:

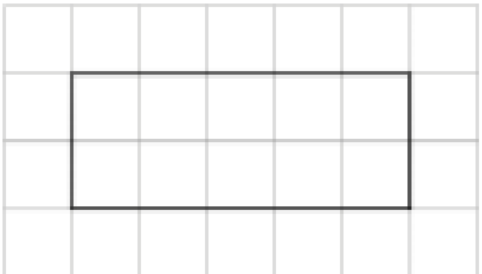


Figure 233: a 5cm by 2cm rectangle on a squared-centimetre grid

Drawn to actual size.

Example 2

Task: draw a pentagon with an area of $10cm^2$.

Example solution:

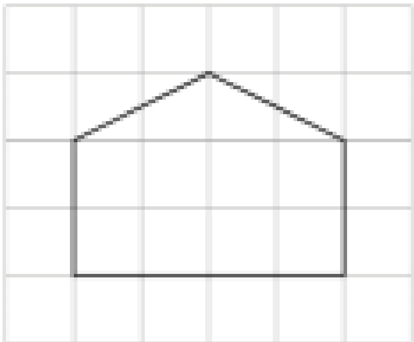


Figure 234: an irregular pentagon with an area of $10cm^2$

Drawn to actual size.

Examples like these involve more than 1 unknown and have more than 1 solution. Pupils should learn to choose a value for 1 of the variables and work out other unknowns from this. For example, to draw a rectangle with a perimeter of 14cm, the width could be chosen to be 1cm and the length then calculated to be 6cm, or the width could be chosen to be 2cm and the length then calculated to be 5cm. There are 6 possible whole-number solutions to this problem (counting the same rectangle in a different orientation as a separate solution), and pupils may provide any one of them to complete the task.

Pupils should be able to reason about dimensions or areas given for part of a shape to determine the values for other parts of a shape or for a compound shape.

Example 3

Problem: find the perimeter of the large rectangle on the right.

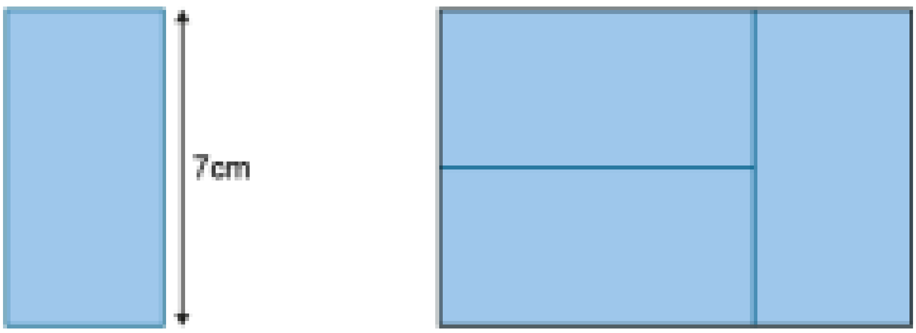


Figure 235: problem involving a compound shape made from 3 identical rectangles

Drawn to scale, not actual size

Solution: perimeter of large rectangle = 35cm

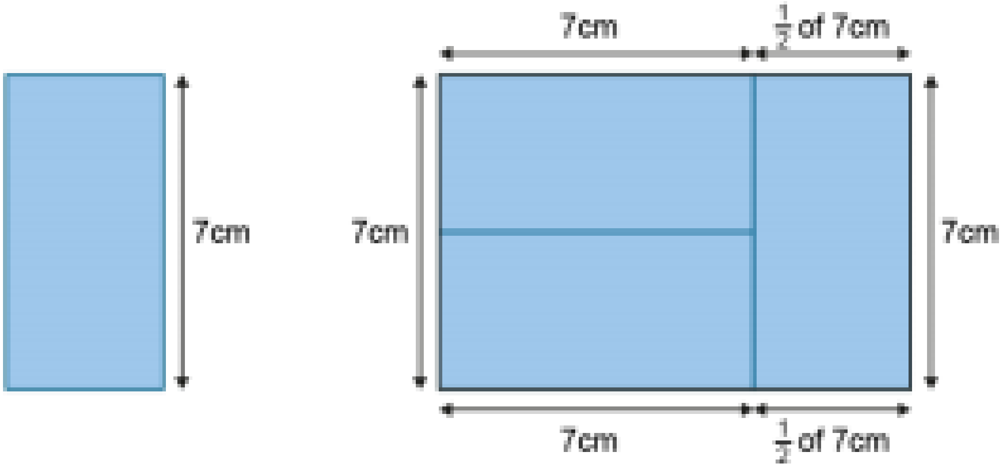


Figure 236 solving a problem involving a compound shape made from 3 identical rectangles

Drawn to scale, not actual size

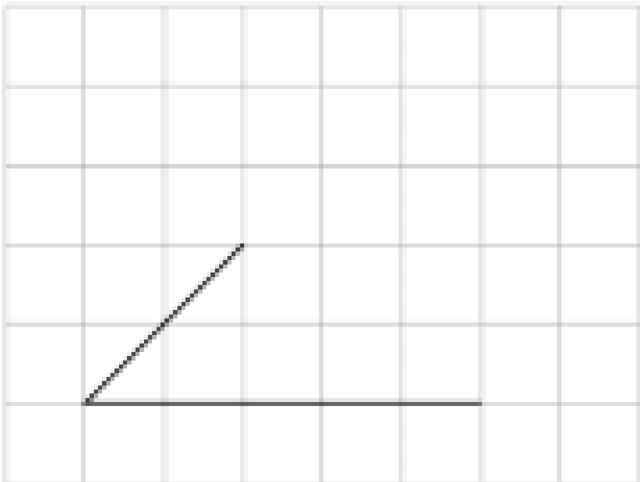
Further examples are provided below in 6G–1, questions 4 and 8.

Making connections

In 6AS/MD-4 pupils learnt to solve problems with 2 unknowns. Drawing a shape to match given properties can correspond to a problem with 2 unknowns (Example 1 above) or more (Example 2 above).

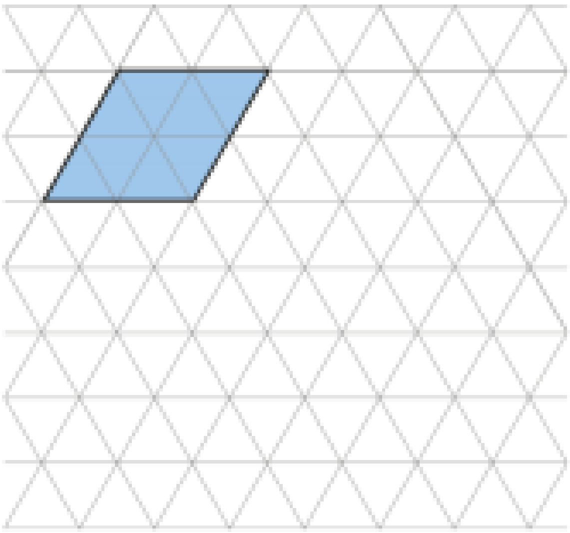
6G–1 Example assessment questions

1. Lois has started drawing a shape on this squared-centimetre grid. Complete her shape so that it has an area of $14cm^2$.

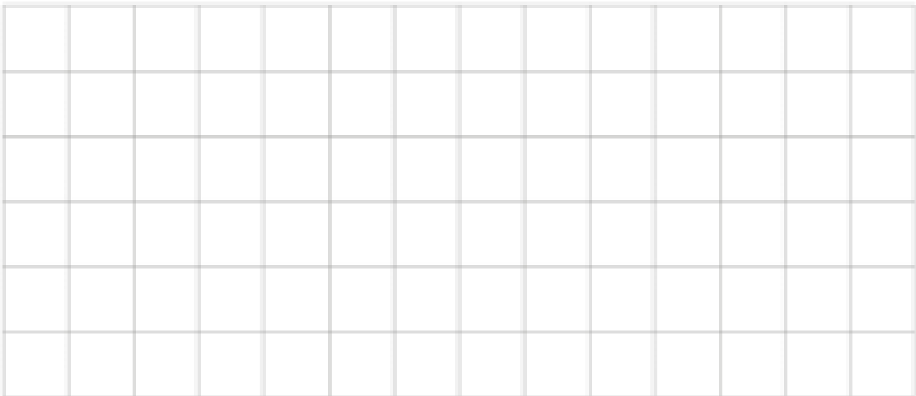


Drawn to actual size

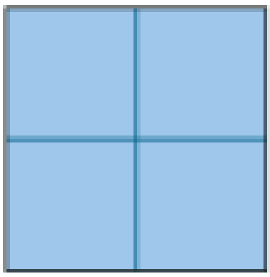
2. Here is a rhombus on a triangular grid. Draw a different shape with the same area on the grid.



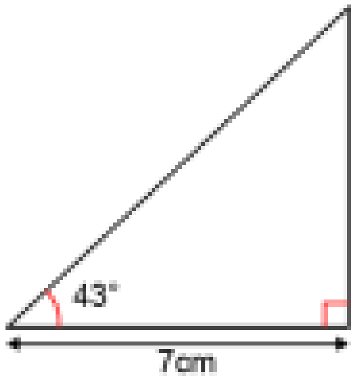
3. Draw a hexagon on this squared-centimetre grid. Include one side of length 4cm and one side of length 3cm.



4. Here is a square made from 4 smaller squares. The area of the large square is $64cm^2$. What is the length of 1 side of each small square?

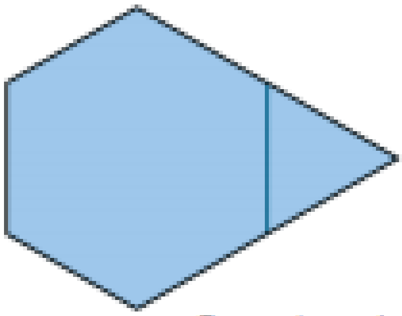


5. Here is a sketch of a triangle. It is not drawn to scale. Draw the full-size triangle accurately. Use an angle measurer (protractor) and a ruler.



Not drawn to scale

6. Here is a picture of a pentagon made from a regular hexagon and an equilateral triangle. The perimeter of the triangle is 24cm. What is the perimeter of the pentagon?



Drawn to scale, not actual size

Calculation and fluency

Number, place value and number facts: 6NPV–1 and 6NPV–2

- **6NPV–1** Understand the relationship between powers of 10 from 1 hundredth to 10 million, and use this to make a given number 10, 100, 1,000, 1 tenth, 1 hundredth or 1 thousandth times the size (multiply and divide by 10, 100 and 1,000).
- **6NPV–2** Recognise the place value of each digit in numbers up to 10 million, including decimal fractions, and compose and decompose numbers up to 10 million using standard and non-standard partitioning.

Pupils should develop fluency in multiplying numbers by 10, 100 and 1,000 to give products with up to 7 digits, and dividing up to 7-digit numbers by 10, 100 and 1,000.

Pupils should be able to carry out calculations based on their understanding of place-value as well as non-standard partitioning, for example:

$4{,}000 + 30{,}000 + 0.2 + 5{,}000{,}000 = \square$

$381{,}920 - 900 = \square$

$518.32 + 30 = \square$

$381{,}920 - 60{,}000 = \square$

Pupils should also be able to apply their place-value knowledge for larger numbers to known additive and multiplicative number facts, including scaling both factors of a multiplication calculation, for example:

$8 + 6 = 14$

$800{,}000 + 600{,}000 = 1{,}400{,}000$

$3 \times 4 = 12$

$3 \times 40{,}000 = 120{,}000$

$300 \times 400 = 120{,}000$

Representations such as place-value counters, partitioning diagrams and Gattegno charts can be used initially to help pupils understand calculation strategies and make connections between known facts and related calculations. However, pupils should not rely on such representations for calculating.

Pupils should maintain fluency in both formal written and mental methods for calculation. Mental methods can include jottings to keep track of calculations. Pupils should select the most efficient method to calculate depending on the numbers involved.

Pupils should learn to check their calculations with a calculator so that they know how to use one. This will help pupils when they progress to key stage 3.

Addition and subtraction: formal written methods

Pupils should continue to practise adding whole numbers with up to 4 digits, and numbers with up to 2 decimal places, using columnar addition. This should include calculations with more than 2 addends, and calculations with addends that have different numbers of digits.

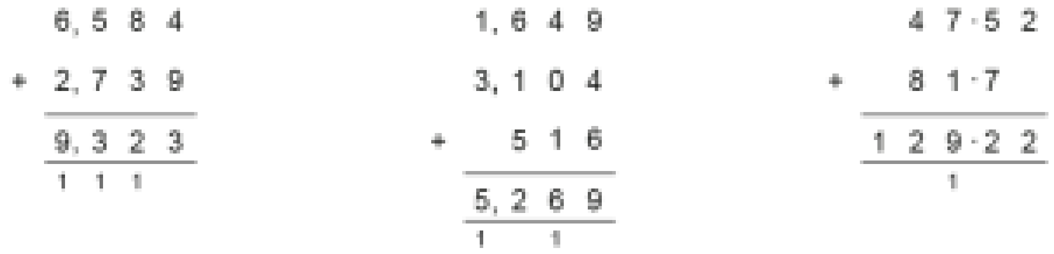


Figure 237: range of columnar addition calculations

For calculations with more than 2 addends, pupils should add the digits within a column in the most efficient order. For the second example above, efficient choices could include:

- beginning by making 10 in the ones column
- making double-6 in the hundreds column

Pupils should continue to practise using columnar subtraction for numbers with up to 4 digits, and numbers with up to 2 decimal places. This should include calculations where the minuend and subtrahend have a different numbers of digits or decimal places, and those involving exchange through 0.

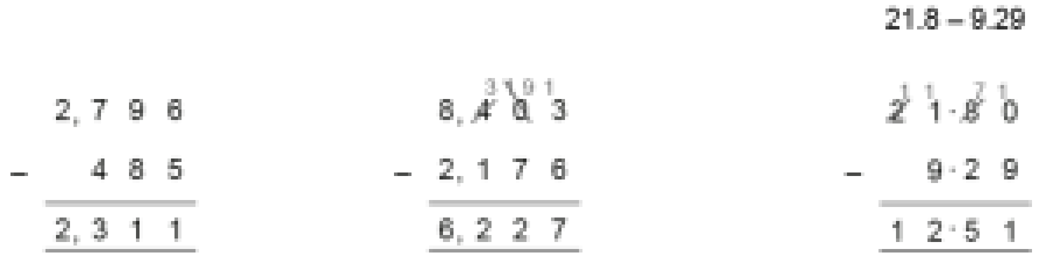


Figure 238: range of columnar subtraction calculations

Pupils should make sensible decisions about how and when to use columnar methods. For example, when subtracting a decimal fraction from a whole number, pupils may be able to use their knowledge of complements, avoiding the need to exchange through zeroes. For example, to calculate $8 - 4.85$ pupils should be able to work out that the decimal complement to 5 from 4.85 is 0.15, and that the total difference is therefore 3.15.

Pupils should learn to check their columnar addition and subtraction calculations with a calculator so that they know how to use one. This will help pupils when they progress to key stage 3.

Multiplication: extending 5MD–3

In year 5, pupils learnt to multiply any whole number with up to 4 digits by any 1-digit number using short multiplication (5MD–3). They should continue to practise this in year 6. Pupils should also learn to use short multiplication to multiply decimal numbers by 1-digit numbers, and use this to solve contextual measures problems, including those involving money.

$$\begin{array}{rr} & 2\ 4 \\ \times & 6 \\ \hline & 1\ 4\ 4 \\ \hline & {\scriptstyle 2}\ \ \end{array} \qquad \begin{array}{rr} & 3\ 4\ 2 \\ \times & 7 \\ \hline & 2,3\ 9\ 4 \\ \hline & {\scriptstyle 2\ \ 1}\ \ \end{array} \qquad \begin{array}{rr} & 5\cdot 3\ 5 \\ \times & 4 \\ \hline & 2\ 1\cdot 4\ 0 \\ \hline & {\scriptstyle 1\ \ 2}\ \ \ \ \end{array}$$

Figure 239: range of short multiplication calculations

Pupils should be able to multiply a whole number with up to 4 digits by a 2-digit whole number by applying the distributive property of multiplication (4MD–3). This results in multiplication by a multiple of 10 (which they can carry out by writing the multiple of 10 as a product of 2 factors (5MD–3) and multiplication by a one-digit number.

$$\begin{aligned} 124 \times 26 &= 124 \times 20 + 124 \times 6 \\ &= 124 \times 2 \times 10 + 124 \times 6 \\ &= 2{,}480 + 744 \\ &= 3{,}224 \end{aligned}$$

Pupils should be able to represent this using the formal written method of long multiplication, and explain the connection to the partial products resulting from application of the distributive law.

		1	2	
		1	2	4
×			2	6
		7	4	4
	2,	4	8	0
	3,	2	2	4
	1	1		

Figure 240: long multiplication calculation

Pupils should be fluent in interpreting contextual problems to decide when multiplication is the appropriate operation to use, including as part of multi-step problems. Pupils should use short or long multiplication as appropriate to solve these calculations.

Pupils should learn to check their short and long multiplication calculations with a calculator so that they know how to use one. This will help pupils when they progress to key stage 3.

Division: extending 5MD–4

In year 5, pupils learnt to divide any whole number with up to 4 digits by a 1-digit number using short division, including with remainders (5MD–4). They should continue to practise this in year 6. Pupils should also learn to use short division to express remainders as a decimal fraction.

5) 4 3 ³2	8) 4, 9 ¹5 ⁷2	4) 1 0 ²9 · ¹0 ²0
8 6 r 2	6 1 9	2 7 · 2 5

Figure 241: range of short division calculations

For contextual problems, pupils must be able to interpret remainders appropriately as they learnt to do in year 4 (4NF–2). This should be extended to making an appropriate decision about how to represent the remainder. Consider the question "4 friends equally share the cost of a £109 meal. How much does each of them pay?" Pupils should recognise that an answer of £27 remainder 1 is not helpful in this context, and that they need to express the answer as a decimal fraction (£27.25) to provide a sufficient answer to the question.

Pupils should also be able to divide any whole number with up to 4 digits by a 2-digit number, recording using either short or long division. Pupils are likely to need to write out multiples of the divisor to carry out these calculations and can do this efficiently using a ratio table – they can write out all multiples up to 10× (working in the most efficient order) or write out multiples as needed.

	×17
1	17
2	34
3	51
4	68
5	85
6	
7	
8	136

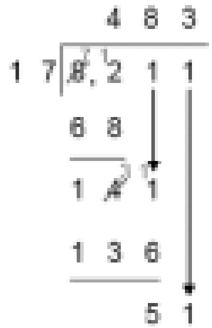


Figure 242: long division calculation ($8{,}211 \div 17$)

Pupils should be fluent in interpreting contextual problems to decide when division is the appropriate operation to use, including as part of multi-step problems. Pupils should use short or long division as appropriate to solve these calculations.

Pupils should learn to check their short and long division calculations with a calculator so that they know how to use one. This will help pupils when they progress to key stage 3.

Appendix: number facts fluency overview

Addition and subtraction facts

The full set of addition calculations that pupils need to be able to solve with automaticity are shown in the table below. Pupils must also be able to solve the corresponding subtraction calculations with automaticity.

+	0	1	2	3	4	5	6	7	8	9	10
0	0 + 0	0 + 1	0 + 2	0 + 3	0 + 4	0 + 5	0 + 6	0 + 7	0 + 8	0 + 9	0 + 10
1	1 + 0	1 + 1	1 + 2	1 + 3	1 + 4	1 + 5	1 + 6	1 + 7	1 + 8	1 + 9	1 + 10
2	2 + 0	2 + 1	2 + 2	2 + 3	2 + 4	2 + 5	2 + 6	2 + 7	2 + 8	2 + 9	2 + 10
3	3 + 0	3 + 1	3 + 2	3 + 3	3 + 4	3 + 5	3 + 6	3 + 7	3 + 8	3 + 9	3 + 10
4	4 + 0	4 + 1	4 + 2	4 + 3	4 + 4	4 + 5	4 + 6	4 + 7	4 + 8	4 + 9	4 + 10
5	5 + 0	5 + 1	5 + 2	5 + 3	5 + 4	5 + 5	5 + 6	5 + 7	5 + 8	5 + 9	5 + 10
6	6 + 0	6 + 1	6 + 2	6 + 3	6 + 4	6 + 5	6 + 6	6 + 7	6 + 8	6 + 9	6 + 10
7	7 + 0	7 + 1	7 + 2	7 + 3	7 + 4	7 + 5	7 + 6	7 + 7	7 + 8	7 + 9	7 + 10
8	8 + 0	8 + 1	8 + 2	8 + 3	8 + 4	8 + 5	8 + 6	8 + 7	8 + 8	8 + 9	8 + 10
9	9 + 0	9 + 1	9 + 2	9 + 3	9 + 4	9 + 5	9 + 6	9 + 7	9 + 8	9 + 9	9 + 10
10	10 + 0	10 + 1	10 + 2	10 + 3	10 + 4	10 + 5	10 + 6	10 + 7	10 + 8	10 + 9	10 + 10

Pupils must be fluent in these facts by the end of year 2, and should continue with regular practice through year 3 to secure and maintain fluency. It is essential that pupils have automatic recall of these facts before they learn the formal written methods of columnar addition and subtraction.

The Factual fluency progression table at the end of this appendix summarises the order in which pupil should learn these additive number facts.

Multiplication and division facts

The full set of multiplication calculations that pupils need to be able to solve by automatic recall are shown in the table below. Pupils must also have automatic recall of the corresponding division facts.

1 × 1	1 × 2	1 × 3	1 × 4	1 × 5	1 × 6	1 × 7	1 × 8	1 × 9	1 × 10	1 × 11	1 × 12
2 × 1	2 × 2	2 × 3	2 × 4	2 × 5	2 × 6	2 × 7	2 × 8	2 × 9	2 × 10	2 × 11	2 × 12
3 × 1	3 × 2	3 × 3	3 × 4	3 × 5	3 × 6	3 × 7	3 × 8	3 × 9	3 × 10	3 × 11	3 × 12
4 × 1	4 × 2	4 × 3	4 × 4	4 × 5	4 × 6	4 × 7	4 × 8	4 × 9	4 × 10	4 × 11	4 × 12
5 × 1	5 × 2	5 × 3	5 × 4	5 × 5	5 × 6	5 × 7	5 × 8	5 × 9	5 × 10	5 × 11	5 × 12
6 × 1	6 × 2	6 × 3	6 × 4	6 × 5	6 × 6	6 × 7	6 × 8	6 × 9	6 × 10	6 × 11	6 × 12
7 × 1	7 × 2	7 × 3	7 × 4	7 × 5	7 × 6	7 × 7	7 × 8	7 × 9	7 × 10	7 × 11	7 × 12
8 × 1	8 × 2	8 × 3	8 × 4	8 × 5	8 × 6	8 × 7	8 × 8	8 × 9	8 × 10	8 × 11	8 × 12
9 × 1	9 × 2	9 × 3	9 × 4	9 × 5	9 × 6	9 × 7	9 × 8	9 × 9	9 × 10	9 × 11	9 × 12
10 × 1	10 × 2	10 × 3	10 × 4	10 × 5	10 × 6	10 × 7	10 × 8	10 × 9	10 × 10	10 × 11	10 × 12
11 × 1	11 × 2	11 × 3	11 × 4	11 × 5	11 × 6	11 × 7	11 × 8	11 × 9	11 × 10	11 × 11	11 × 12
12 × 1	12 × 2	12 × 3	12 × 4	12 × 5	12 × 6	12 × 7	12 × 8	12 × 9	12 × 10	12 × 11	12 × 12

Pupils must be fluent in these facts by the end of year 4, and this is assessed in the multiplication tables check. Pupils should continue with regular practice through year 5 to secure and maintain fluency.

The 36 most important facts are highlighted in the table. Fluency in these facts should be prioritised because, when coupled with an understanding of commutativity and fluency in the formal written method for multiplication, they enable pupils to multiply any pair of numbers.